

Reviewed by David Fowler
University of Nebraska–Lincoln

Sergei Abramovich is a professor in the Department of Curriculum and Instruction at the State University of New York–Potsdam. He holds a Ph.D. in mathematics from the St. Petersburg State University in Russia and has had appointments with the Department of Mathematics Education at the University of Georgia and the Department of Mathematics, Statistics, and Computer Science at the University of Illinois-Chicago.

The collection of topics included in this book could provide a solid mathematical background for a prospective elementary teacher or for a current teacher wishing to develop more mathematical

knowledge. As important as the set of topics is the narrative that runs through the text.

Abramovich consistently shows how to think mathematically, using a variety of tools and representations. The book uses spreadsheets, computer geometry, and elementary graphics software to investigate problems, although much of the essential mathematical content can also be studied using traditional hand-and-mind-based calculation. An instructor could apply the technology available in her or his teaching environment and adapt the lessons to the tool set of choice. Instead of The Geometer's Sketchpad, for example, one could use the recently developed (and free) GeoGebra. The particular technology is less important than the ways the technology is introduced to the students and used to convey mathematical meaning. Properly used, this book will help develop the mathematical habits of mind and habits of pedagogy that are fundamental to good mathematics teaching.

The chapters in Abramovich's book focus on thinking that is grounded in basic mathematics. He often begins with simple problems an elementary student might encounter, and then extends these problems into serious mathematical discussions. A thoughtful education student, with the guidance of a mathematically and pedagogically informed instructor,1 will see repeatedly that mathematics, even at the elementary level, is not just a sequence of computational algorithms. The first chapter includes a perfect illustration of this, with an exploration of the integer partition problem.

The partition problem is easily stated: How many ways can you represent a whole number as a sum of smaller whole numbers? The number 4, for example, can be written simply as 4, or as 3+1, 2+2, 2+1+1, or 1+1+1+1 — in all, five different ways, not counting different ordering

1 Ideally, an instructional team would include a mathematician and an expert on the cognitive development of children. See, for example, "A Mathematician–Mathematics Educator Partnership to Teach Teachers." Ruth M. Heaton and W. James Lewis. Notices of the American Mathematical Society. 58:3. March 2011.
of the terms. This simple fact may not seem to hold much mathematical excitement, but the search for patterns in the partition of integers has challenged mathematicians of the highest order, including the unique Srinivasa Ramanujan, whose fascination with the partition problem is poignantly described in David Leavitt's novel, *The Indian Clerk*.

Abramovich builds a series of mathematical questions around partitions of integers, and shows—as he does throughout the book—how to use physical objects, diagrams, spreadsheets, and formulas to support the investigations. He also includes examples of young children's work, to help students see the connection between the mathematical understanding of the child and the child's adult teacher. By the end of this section, a student should begin to understand that the partition problem, especially when one looks at larger numbers, does not have a simple solution.

Abramovich includes topics dealing with the structure of mathematics, such as the Euclidean Algorithm, and with the cultural history of mathematics, for example, the Fibonacci numbers and the "Golden Ratio." He shows the recursive definition as well as the closed-form expression for the nth Fibonacci number: \( \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2\sqrt{5}} \), and explains the convergence of the Fibonacci sequence to the ratio \((1 + \sqrt{5})/2\).

He includes a chapter on proof and proving that offers an explanation of mathematical induction, with exercises to develop an understanding of the induction principle. An additional strength of the book is the integration of geometric thinking with arithmetic examples. The topic of geometry was long neglected in elementary school—the professional journal for elementary teachers until the early 1990s was called "The Arithmetic Teacher." Abramovich includes topics on application of unit fractions to plane tessellations, and explorations with polygonal numbers; for future elementary teachers, these are valuable examples of the connections between numbers and geometric objects. His many examples using geometric representations for fractions should help disabuse elementary teachers of their preoccupation with describing fractions using "round food" (pizzas, cookies, pies, ...) as Liping Ma wrote in her book, *Knowing and Teaching*
In summary, this book addresses both "what topics should elementary teachers know" and "how should elementary teachers think about mathematics?" I'll address the difference with an example from my own observation of a graduate student in mathematics working with pre-service elementary teachers. Mathematicians are fond of using combinatorial problems as examples of "real world" mathematics. A simple, but typical example is the following problem: how many ways can five different books be arranged on a shelf? Although books on a shelf are "real world" objects, the group of future elementary teachers the student was working with saw this as a pointless question. Their opinion was that at most a half-dozen arrangements were of any interest: alphabetical by title, alphabetical by author, ordered by library catalog number, possibly grouped by subject, and so forth. When told there could be 120 different arrangements, their response was irritation. One student saw the problem as a "trick question."

Another student, trying to connect this with actual media center practice, said that it just showed that children should not reshelve the books. The intended lesson—the rapid growth of the factorial function—was lost in the discussion by a group of students who could, if required, learn a permutation formula, but would not willingly ponder the formula's mathematical implications. Studying this book gives me the sense that Abramovich has had better success in getting future teachers to think mathematically.

Criticisms of elementary mathematics education in the United States occur periodically, like cicada emergences, and create at least as great a racket. Recent contributions to the buzz include the 2010 publication *Rising Above the Gathering Storm, Revisited: Rapidly Approaching Category 5*, published by the National Academy of Sciences (NAS). The NAS publication includes a severe report on the preparation of elementary teachers:

... Sixty-nine percent of United States public school students in fifth through eighth grade are
taught mathematics by a teacher without a
degree or certificate in mathematics.

Although the meteorological metaphor might suggest that,
once again, despite the talk, nobody will do anything about
the weather, steps are underway to enact educational
climate change. A significant effort at the time of this
review is the Common Core State Standards Initiative
(CCSSI): "The Standards for Mathematical Practice
describe varieties of expertise that mathematics educators
at all levels should seek to develop in their students." The
book by Sergei Abramovich provides an excellent tool for
preparing teachers who can develop mathematical
expertise in their students.

About the Reviewer

David Fowler is a professor of mathematics
education in the College of Education and Human
Sciences, with additional appointment in the
Department of Mathematics, at the University of
Nebraska-Lincoln. He can be reached at
dowler1@unl.edu.