Education Review Reseñas Educativas
Resenhas Educativas

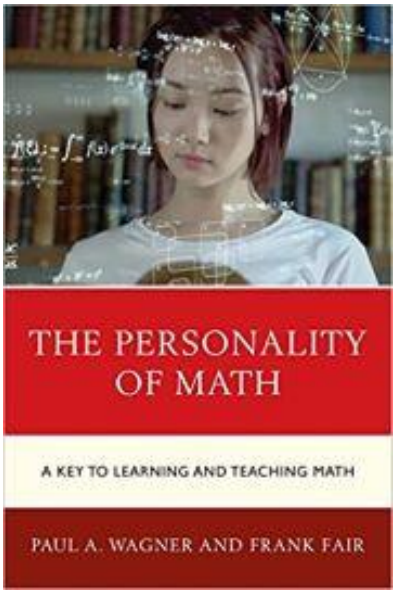
March 29, 2023 ISSN 1094-5296

Wagner, P. A., & Fair, F. (2022). *The personality of math: A key to learning and teaching math.* Rowman and Littlefield.

Pp. 152 **ISBN: 978-1475862973**

Reviewed by David Fowler
University of Nebraska, Lincoln
United States

In January 2018, the American Mathematical Society introduced a colorful and modern-looking image to serve as the official seal of the society. The previous image—a stylized Grecian temple with Greek characters stating "No entrance without knowledge of geometry" was discarded when recent historical research raised doubts about the story that Plato's Academy bore this motto above its entrance. Furthermore, market research showed that the younger members of the society associated a classic Greek temple with a financial institution, not with mathematics (Ribet, 2018). Thus, even among mathematicians, an older mindset sometimes needs to be replaced with a newer one.



Among non-mathematicians, an image most likely associated with mathematics would be a chalkboard covered with obscure symbols. For some, whether mathematicians or not, Bertrand Russell's (1960) description might invite, or discourage, further interest:

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely

Fowler, D. (2022, March 29). Review of *The personality of math: A key to learning and teaching math* by P. A. Wagner & F. Fair [Book review]. *Education Review*, *30*. https://doi.org/10.14507/er.v30.3693

> pure, and capable of a stern perfection such as only the greatest art can show. (p. 14)

The Personality of Math by Paul A. Wagner and Frank Fair attempts a far different view of mathematics. They perceive mathematics as an entity with a positive, if somewhat mysterious, "personality." In describing David Hilbert's illustration of a countably infinite set, they write:

> You may well ask, who dreams this stuff up? Mathematicians do. They can be a fun group of people imagining things no one has ever seen portrayed on television or anywhere else. Mathematicians love math and have fun teasing her with new tickles, hoping to reveal more of her mystery. Math has nooks and crannies inviting those who become her lovers and heroes. (p. 25)

Wagner and Fair have reviewed the historic record of the past two millennia looking for these lovers and heroes, using multiple sources to construct their image of mathematics. To create the "personality features of math itself," Wagner and Fair assembled a composite personality from the numerous biographies and anecdotes that form major sections of their book. The result seems something like a photomosaic—an image created from multiple reduced images. There is no physical image here, of course; no Russell's statue, cold and austere. Wagner and Fair give us a conceptual entity that uses the pronouns "She/ Her/Hers." In place of multiple graphic images forming a composite image, these authors assemble an abstract personality construct.

> Like a human being with a distinctive personality, the characteristics of the personality of math place certain requirements on those who would truly get to know it. Five of the most important of these are that the potential friend:
>
> - must be completely *honest* in whatever claims are made,
> - must have respect for the existence of truth beyond approximation, *truth that is exact*,
> - must be committed to displaying *the certainty* that math can achieve, not settling for anything less,
> - must work to display truth by producing *proofs* that survive scrutiny and statements that are *free of contradictions*, and
> - must recognize that, while math is rooted in the common world of counting, often for purposes of trade, and holds inestimable value enabling humans to decipher patterns in visible nature, *math's objects*—squares, prime numbers, and all the rest—*are not built out of the material of human sensory observation alone* (p. 49)

Wagner and Fair add to their description of the personality with repeated references to mathematics as a "wilderness." They refer to the historical and

contemporary figures they've selected for the composite personality as "mathematical heroes [who] are adventurers into the mathematical wildernesses." This metaphor becomes the basis for the subtitle of their book: *A Key to Learning and Teaching Math*. They specify that "A curricular and instruction path must be created that invites students who are potential heroes to experience deeper visions into mathematical passion" (p. 9).

They intend for that path and instructional approach to result in an alignment of a student's personality traits with "the personality of math itself" (p. 2). This alignment is to be accomplished, in part, by use of a "Threshold" concept developed in previous work by one of the authors (Wagner, 2021).

Chapter One of T*he Personality of Math* includes a summary of that pedagogical approach. The Wagner and Fair instructional model begins with a period of basic instruction in math facts, perhaps also with some hands-on manipulative work. They call this "indoctrination," justifying the term with a short etymological disclaimer. They also refer to standard classroom practice as "ritualized signaling," borrowing the phrase from sources in evolutionary biology. This indoctrination phase should be periodically interrupted by "benchmarking as to whether students are developing a feeling for and an understanding of the beliefs and dispositions they are acquiring" (p. 4). Given sufficient indoctrination on a given area of knowledge, the student can be seen as reaching a threshold, beyond which further indoctrination on that specific bit of knowledge is contraindicated. Once the threshold is "breached," as they call it, "the student can use the indoctrination to express doubts, ask questions, seek further insights; in short, hopefully to engage in real mathematical thinking" (p. 16).

Their suggested pedagogy can be seen as a recursive, dialectical process. The expression of "well-reasoned doubt" is essential:

> More than any other sort of evidence, it is the expression of reasoned doubt that benchmarks successful passage through a threshold. When the threshold has been successfully breached, further indoctrination is no longer warranted. Learners are finally engaged in the Great Conversation of Humankind just as they should be, and some are on their way to becoming heroes of mathematics. (p. 16)

This leads to a further theme of the book that teachers will need to look at carefully: "there are distinctions between being 'good at math,' being 'very good at math,' and being an actual 'champion of math'" (p. 30). With this theme, the authors take on two grand challenges: How can math education be improved? What is mathematics? I believe the authors achieve partial success on the first challenge. As to the latter challenge, the book could be added to the library of similar attempts as a somewhat uneven beginner's introduction to the history and culture of mathematics.

One can posit interesting thought experiments about teaching, imagining for example, that the pressures exerted by testing based on state or nationally recommended standards like the Common Core could be eased. The approach to learning outlined by the authors could be useful to a home-schooling parent or a private math tutor. These "thresholds," like "teachable moments," can't be specified in a curriculum guide. They occur at different points in each student's development and must be individually identified. Details of how this individualized instruction would work in a typical school setting at each grade level are not included in their discussion.

Many educators are looking for ways to help all students gain basic knowledge that will help them adapt to changing workplace needs. Teachers will be more concerned with alignment of a student's personality to collaborative work with colleagues than with alignment to a legendary hero. The authors apply the term "personality" to students at any grade level, to living and historical figures, and to their constructed personality of math. This leads to very broad generalizations—about students and about mathematics. For the purposes of this book, a conversational, or "folk psychology" understanding of personality traits may be adequate, although a school psychologist might want more gradation. Note, for example, that the use of the expression "idiot savant" (p. 53) is now considered dated and offensive.

One of the authors' summative recommendations is that all teachers present the subject with the zeal and love for mathematics shown by individual heroes in the Wagner and Fair catalog. Educational programs that give the teachers the chance to study real mathematics with inspiring mathematicians can help in this regard, especially if the mathematics is informed by current practice, which is highly collaborative. Although a mathematician may present work done individually to a journal, other mathematicians must serve as referees to guarantee the validity of the submission, even if the submission comes from a mathematical luminary like Marcus Du Sautoy. Mathematics departments regularly hold seminars and colloquia at which new ideas are presented and critiqued. Mathematicians are given sabbaticals and awarded additional research time for their "journeys into the wilderness," but it is a mistake to view their work as a solo heroic journey. An excellent example of this is the recent book *Count Me In: Community and Belonging in Mathematics*. The book is reviewed in the March 2023 *Notices of the American Mathematical Society*, an issue paying special attention to Women's History Month. The review, by Emily Olson (2023) includes this relevant passage:

> What are the biggest open problems in your area of mathematics today? Perhaps your first thought was something quite famous, such as the Riemann hypothesis or P versus NP. Perhaps you have an open problem in your area of mathematics that you have thought about for a few months (or a few years) There are very few open problems that affect almost every

> mathematician. *Count Me In* highlights one in particular: How can we increase diversity among mathematicians? (p. 464)

The Socratic approach of indoctrination-threshold/benchmark-doubt used to create a champion of math might fit into the excellent national math competitions in which many schools participate. Finding the individual benchmark thresholds for the 100 or more students a secondary public school teacher meets in a day would be an instructional challenge in itself. Practical application aside, terms in this book that would certainly hinder the adoption of the pedagogy described include the use of "indoctrination"—despite their etymological disclaimer—to describe the presentation of mathematical ideas in a classroom. Teachers, who today are fighting charges of indoctrination from anti-education extremists, will not want that word included in their lesson plans. The reference to standard teaching as "ritualized signaling," using the referenced book on animal communication to describe math classroom discourse, would not be well received (pp. 12 ff.). A school psychologist's view of "personality" would not be the same as the "personality" of math constructed from the "cherry picking" methodology described by Wagner and Fair (p. 40). Parents would have a different view of their child's personality than the ones described in many of the examples included in the book. Finally, promoting "doubt" and "aligning the personality of the student" could quickly be dangerously misinterpreted. One hopes not to witness our teachers meeting the same fate, even figuratively, of Socrates or Hypatia.

The most serious educational flaw in the Wagner and Fair scheme is their representation of mathematics as a female entity pursued by *suitors*, a term that in any standard definition refers to men trying to win the affection of a woman with a hope of future marriage. I won't belabor this point, other than to say that including information on a few women mathematicians, such as the remarkable Emmy Noether, does not offset statements such as "These challenges reveal mathematics as truly the queen of all searches for reality" (p. 16); "Personality of the heroes and their marriage with math" (p. 41); or "Time to take a look at the human side of math's personality by looking toward some of her most famous suitors" (p. 57).

Writing about the history and development of any mathematical topic can be a daunting endeavor. The term "math" is often modified as "school math," considered a separate subject from higher level mathematics (Gold, 2017). In fact, even at the research level, there is still differentiation among the highest levels of mathematical creativity (Zalamea, 2019). Mathematics concepts tend with time to descend through the hierarchy. A prime example of this descent is the calculus notatation introduced by Gottfried Leibniz in a scientific journal in 1682. That notation is now routinely used in high school introductory calculus courses (Antognazza, 2011). In this sense, "mathematics" can eventually become "math."

Wagner and Fair understand that from a philosophical point of view, simply recounting facts of the discipline's development is not enough. In his essay on the origin of geometry, by which he meant all disciplines that deal with shapes existing mathematically in pure space-time, Edmund Husserl wrote: "… to cling to this merely historically factual aspect of mathematics is precisely to lose oneself to a sort of romanticism and to overlook the genuine problem, the internal-historical problem, the epistemological problem" (Husserl, 1978). Wagner and Fair have taken on the task of finding an internal story that runs through the historical facts of mathematics, and they present this story based on character traits selected from more than two dozen mathematical personages ranging from Pythagoras to Gregori Perelman. Their book interpreted as an extended literary gloss on this story might be a useful addition to a math teacher's library, especially with the detailed bibliography. A few errors should be corrected, and their examples of "threshold" conversations could be enhanced with a more mathematical treatment.

A useful correction regarding "Fibonacci numbers" would follow from Keith Devlin's (2017) account of Leonardo of Pisa (p. 45). Devlin shows that Leonardo did not explore phyllotaxis or the other applications of "Fibonacci numbers" attributed to him. All that came about 600 years later, in the work of Auguste Bravais. Students and teachers should be cautioned to be aware of the Rashomon Effect in trying to decide with certainty the "true" personality of an individual. At one point in this book, Ada Lovelace is described as "no more than a groupie of the intellectual elite" (p. 57). Elsewhere we read "her love of math stayed with her until the end of her days" (p. 101).

Other material from history needs to be amplified, either by looking up references in the bibliography or finding legitimate information through intelligent Internet searches. There is also the danger that famous "thought experiments" often make more sense to experts who already know something about the context of the story than to beginners or outsiders. Upon hearing that Achilles can't catch the tortoise or that the Hotel Hilbert can always accept another (countable) infinity of new guests, the math novice may well form a deflationary reduction of the story. Typical presentations of the infinite hotel, in particular, could lead children to imagine David Hilbert as a sort of Hans Christian Anderson for mathematics whose fame rests on his parables: See the paper by Helge Kragh (2014), "The True (?) Story of Hilbert's Infinite Hotel." Zeno's paradoxes, as interpreted by Socrates, may have been intended as parody. These examples require additional reading from the choices listed in the bibliography. It would be helpful to know if the imagined conversation between student and teacher would be left with the conclusion that an infinite collection of universes must contain identical universes. (p. 15)

Although the book is not intended as a mathematics text, some examples of "mathematical thinking" could be improved. For example, the example of finding an approximation to the square root of 2 by using decimals—1.41 and 1.42—would benefit from starting with some fractions, such as 3/2, 7/5, and 17/12. The student could see the squares of these values are getting closer to 2, and with some work even find a pattern for continuing the sequence. At the same time, the fact that these ratios are not exactly 2 would help the student understand how "not a ratio" became "irrational"—a word that most people think of in terms of crazy behavior. The discussion of zero in the book may illustrate in a vague way the reluctance of early arithmeticians to consider zero as a number. A simple explanation by an elementary teacher using the numbers 11 and 101 shows clearly that there is "nothing" between the 1s in the first number and "zero" between the 1s in the second. This concise example also can lead to an understanding that zero is used to establish our system as not only a base ten system, but a place value system as well. The unique properties of zero discussed on page 88 do show that zero has its own identity, in fact it is called an additive identity in advanced algebra. The younger student, given this "indoctrination" fact, could then be asked if there could be more than one identity in arithmetic, which could open the door to a discussion of mathematical proof.

In assembling their collection of facts, and in threading their own inner story through this collection, Wagner and Fair have come up against the semanticist's Ladder of Abstraction. As the mathematician Andrew Gleason (1969) summarized in a review of the evolution of differential topology:

> I have discussed this general field of mathematics entirely in terms of specific facts and problems. Unfortunately the methods involved in proving these facts involve such a long journey up the ladder of abstraction that it is impossible to give, in any brief article, a fair idea of how they work. Separated from the methods which establish them, fact can convey only a partial picture of mathematics. Understanding these methods is reserved for those who devote years to the study of mathematics (p. 188).

Wagner and Fair use their own ladder of abstraction in the final chapters of the book, explaining their picture of mathematics with reference to "The Great Conversation of Humankind," and "The Law of Figuring Things Out." These ideas are explored and explained in more detail in their book *Thinking Ahead: Engaging all Teachers in Critical Thinking* (Wagner et al., 2018). This and other previous writing by the authors cited in the bibliography will be helpful to someone wanting to understand their philosophic approach to mathematics education.

References

Antognazza, M. R. (2011). *Leibniz: An intellectual biography*. Cambridge University Press.

Devlin, K. (2017). *Finding Fibonacci: The quest to rediscover the forgotten mathematical genius who changed the world*. Princeton University Press

Gleason, A. (1969). The evolution of differential topology. In G. A. W. Boehm (Ed.), *The mathematical sciences. A collection of essays.* MIT Press.

Gold, B. (2017). School mathematics and "real" mathematics. In B. Sriraman, (Ed.), *Humanizing mathematics and its philosophy* (pp 125-137). Springer. https://doi.org/10.1007/978-3-319-61231-7_12

Husserl, E. (1978). The origin of geometry. In J. Derrida, *Edmund Husserl's origin of geometry — An introduction*. Nicolas Hays, Ltd.

Kragh, H. (2014). *The true (?) story of Hilbert's infinite hotel.* Arxiv – Physics. https://doi.org/10.48550/arXiv.1403.0059

Olson, E. (2023). Book review of *Count Me In*. *Notices of the American Mathematical Society*, *70*(3), 464.

Ribet, K. A. (2018). Inside the AMS. *Notices of the American Mathematical Society*, *65*(1), 69.

Russell, B. (1960). The unreasonable effectiveness of mathematics in the natural sciences. Richard Courant lecture in mathematical sciences delivered at New York University, May 11, 1959. *Communications on Pure and Applied Mathematics*, *13*(1), 14.

Wagner, P. A. (2021). The methods, benefits and limitations of indoctrination in mathematics education. *Interchange: A Quarterly Review of Education*, *52*(1), 41–56.

Wagner, P. A., Johnson, D., Fair, F., & Fasko, D. (2018). *Thinking ahead — Engaging all teachers in critical thinking*. Rowman & Littlefield.

Zalamea, F. (2019) *Synthetic philosophy of contemporary mathematics*. Sequence Press.

About the Reviewer

David Fowler is an Emeritus Professor of Mathematics Education at the University of Nebraska-Lincoln (UN-L). He has an undergraduate degree in mathematics from Harvard College (1962) and PhD from UN-L in mathematics education and instructional technology (1991). His primary interests are in finding uses for emerging technologies in mathematics education, especially with the application *Mathematica*, developed by Wolfram Research, Inc. He has been a Wolfram Visiting Scholar at the WRI campus and was Editor-in-Chief of the journal *Mathematica in Education and Research* (Springer Verlag) from 1998 to 2002. At UN-L he was chair or co-chair for 19 doctoral students and 74 master's students in mathematics or technology education. In 2011, he received the Milton Beckman Lifetime Achievement Award for Advancement of Mathematics Education from the Nebraska Association of Teachers of Mathematics.

Education Review Reseñas Educativas
Resenhas Educativas