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The National Council for Teachers of Mathematics set the stage for equitable mathematics learning for all students in *Principles to Actions: Ensuring Mathematical Success for All* (2014). Teachers are challenged to provide learning experiences so that every student has the support necessary to perform high-quality mathematics work. Understanding how students think about algebraic concepts can arm teachers with the necessary knowledge to make this type of learning a reality for all students. In an effort to provide teachers with information necessary to support algebraic thinking, Rhine, Harrington, and Starr have synthesized 35 years of research on algebraic thinking to produce *How Students Think When Doing Algebra*. These authors have a variety of classroom experiences, from teaching high school and college mathematics to teaching pre-service mathematics teachers to providing professional development to mathematics teachers. They have also studied and researched mathematics education. Their experience with teachers and teaching has helped shaped this book into a usable text for in-service teachers.

In this practitioner-focused guide to algebraic thinking, teachers, instructional coaches, and professional development providers will learn why students’ algebraic thinking makes sense to them and how to improve instructional practices to minimize student misconceptions. The book is a research-based, comprehensive examination of algebraic thinking and teaching practices that will provide all stakeholders with the necessary information to support students in engaging in meaningful algebraic work. I believe teachers who engage with this text will deepen their understanding of algebraic thinking and, hence, improve their practice so that all students have rich mathematical learning experiences.

Although this text was written to support inexperienced teachers, experienced teachers and other professionals will also find it a useful tool in their continued development. The authors argue that teachers must understand the multiple ways students think about and struggle with mathematics in order to make sound pedagogical decisions to help students develop conceptual understanding of mathematics. They suggest that novice teachers “lack the experience to anticipate important moments in their students’ learning” (p. 3). Hence, this text was written to support inexperienced teachers’ learning of students’ common errors, misconceptions, and struggles while doing algebra. Experienced teachers will also find the pedagogical content knowledge discussed rich and thought-provoking. Instructional coaches and professional development providers will appreciate talking points that can be used to expand teachers’ repertoire of instructional strategies.

Improving teachers’ pedagogical content knowledge will facilitate the decision-making processes teachers use daily in their work with students. The decisions teachers make when responding to student thinking impact how students learn mathematics. These “Mathematically Significant Opportunities to build on Students’ Thinking” (MOSTs) (Leatham, Peterson, Stockero, & VanZoest, 2015) are the connections between mathematics, pedagogy, and student thinking. These instructional decisions can impact the MOSTs within classroom structure.

The introduction is a compelling read, as it provides a great deal of context for this topic. The authors base their work on research about why algebraic thinking is vital for all students to learn and why algebra is difficult for students to understand. The introduction details research in Piagetian theory, Dweck’s fixed and growth mindset theory, and the algebraic habits of mind and how these theories apply to algebraic thinking. In particular, they highlight connections between the research and how students develop algebraic reasoning, how students develop misconceptions, and how students think about specific concepts in algebra.

The organization of the book is structured to provide an easy reference for teachers and other professionals as they plan for their algebra course. The authors designed this book with the goal of enhancing teachers’ pedagogical content knowledge so that instructional decisions can be made to improve MOSTs. Each chapter is devoted to an algebraic category: variables and expressions, algebraic relations, analysis of change, patterns and functions, and modeling and word problems. Within each chapter, the authors begin by offering teachers a research-based description of the algebraic concept that challenges and deepens pedagogical content knowledge. They offer reasons why students struggle with each algebraic concept, ranging from the nature of the algebra to common teaching practices that often contribute to students’ misunderstandings. The chapters are then divided into subsections and referenced to specific Common Core State Standards. Each subsection has a consistent framework so that teachers can easily locate information. First, they identify the thinking centered around the algebraic concept and what the
algebraic misconceptions look like symbolically. Second, they specify how students think about the algebraic concept and the misconceptions. Third, they discuss the mathematics in relation to why the misconceptions may occur. Finally, they offer specific instructional moves that could help students develop a deeper understanding of the mathematical idea.

In the discussions, the authors clearly demonstrate algebraic thinking and the related misconceptions. It is noteworthy that they examine algebraic thinking centered around paper-and-pencil tasks as well as the role technology plays in both helping and hindering algebraic thinking. In addition to this framework, the authors provide a comprehensive list of the research at the end of each chapter so that teachers can reference an original source for further reading. The design of the text allows teachers to read cover-to-cover for a comprehensive overview, read a chapter for a deeper understanding of a broader topic, or home in on a specific subsection for an in-depth examination.

At the core of book are the specific, practical examples to improve instructional practices. In each subsection, the authors examine student thinking, offer specific suggestions for teacher planning, and suggest overarching instructional strategies. For example, they offer concrete examples of student thinking from interviews, examination questions, and problem-solving situations. Comparing and contrasting the reader’s students’ thinking to the thinking summarized in each section can offer insights into instructional moves and questions that can be used to rectify misconceptions or further student thinking.

At times, the text lapses into dense language and definitions rather than practitioners’ language. Because of the research-based discussions of algebraic thinking, a new teacher may find many explanations to be overwhelming and difficult to understand. However, novice teachers can use this information as an overview as they begin to prepare for each new unit in algebra. As they begin to accumulate more examples of student thinking, they can go back and reread the text to find suggestions for correcting student misconceptions. New teachers can also use the information to gain insight into their students’ thinking as they begin their career.

All teachers have wondered “What in the world where they thinking?” as they examined students’ work, read students’ explanations, or listened to students’ justifications. How Students Think When Doing Algebra offers useful insights as to why students believe their thinking makes sense and how teachers can mitigate algebraic thinking misconceptions in their teaching. I recommend a three-pronged reading approach to this book for teachers, instructional coaches, and professional development providers. In the first reading, look for ways to recognize and correct student misconceptions in your teaching moves. In the second reading, align your instructional strategies and lesson plans to reflect the recommendations. In the third reading, be aware of how the research can inform your practice. It can be used to home in on specific trouble spots in a groups’ algebraic thinking, provide a holistic overview of the teaching and thinking of algebra, and be a jumping off point to start conversations about how to implement change in the teaching and learning of algebra. I believe all teachers can find gems of information in this book that will improve instructional practices so that all students develop strong algebraic thinking, make connections between representations, and have a deep conceptual understanding of algebra.
References


About the Reviewer

Amy Kassel is currently the Secondary Mathematics Curriculum Coordinator and mathematics instructional coach for Laramie County School District #1 in Cheyenne, Wyoming. She is a National Board Certified teacher and taught high school mathematics for 26 years. She has completed the Math Teacher Leadership program at the University of Northern Colorado, earned the Arch Coal Teacher Achievement Award, and is the recipient of the 2017 Presidential Award for Excellence in Mathematics and Science Teaching. She earned her B.S.Ed. in Mathematics from Southeast Missouri State University, M.S. in Mathematics from Western Illinois State University, and is currently a Ph.D. student in Mathematics Education at the University of Wyoming. She can be contacted at amy.kassel@laramie1.org